CHAPTER 13

Revealed Preference and Model Theory

In this book, we have studied the concept of empirical content in disparate environments. To conclude our study, we wish to suggest that there is a unifying theme behind these exercises. The idea of the empirical content of a theory as *the set of all falsifiable predictions of the theory* is generally applicable, and subject to formal study.

A theory can make predictions which are non-falsifiable. A case in point is the theory of representation by a utility function. Recall Theorem 1.1. The theorem implies that if a preference relation $\succeq$ over $\mathbf{R}^n_+$ possesses a utility representation, then there is a countable set $Z \subseteq \mathbf{R}^n_+$ such that for all $x, y \in X$ for which $x \succ y$, there exists $z \in Z$ for which $x \succeq z \succeq y$. This implication of the theory of utility is not falsifiable. To demonstrate that the theory has been falsified, one would need to establish the non-existence of such a set Z. Doing so involves checking, one-by-one, every possible countable subset Z of $\mathbf{R}^n_+$, a task which can never be completed.

A first and basic issue in understanding empirical content has to do with universal vs. existential axiomatizations. The idea was already introduced in Chapters 9 and 12, where we saw the removal of existential quantifiers as a source of testable implications. The issue of universal and existential axioms goes back to Popper (1959), who thought that a theory with a *universal* description is falsifiable, while an *existential* theory is not.

Popper offers the example of the theory that claims “all swans are white.” This theory is universal, in the sense that it states a property of all swans, or “universally quantifies over swans.” It is easy to see that, in principle, such a theory can be falsified by finding a single swan that is not white. Contrast with Popper’s example of an *existential* theory: that “there exists a black swan.” The existential theory cannot be falsified. Falsifying the theory would involve collecting all possible swans and verifying that each one is not black. We could only do this if we could somehow be sure to have exhaustively checked all the swans in the universe.

Universality is clearly important for falsifiability, but there is a second component that is particularly relevant for the subject of this book. Popper’s idea captures the notion of empirical content and falsifiability very well in many environments. However, economic theories and data are often burdened by *partial observability*. Implicit in Popper's examples is the idea that when we observe a swan, we observe its color. And indeed, in the swans example, this is entirely natural. With economic choice data, on the other hand, it is perfectly reasonable to observe objects but not all their properties. It is common, for example, to observe a pair of alternatives and not be able to observe which of the pair an individual chooses, or would choose. Think of choice from a budget B. When $x \in B$ is chosen this reveals something about the pairs (x,y) for $y \in B$, but nothing about the pairs (z,y) for $z \neq x$. This issue has been important in many of the results we have established. It lies behind the non-testability of concave utility in Afriat's Theorem, for example.[1]

Thus, in contrast to the theories that Popper had in mind, economic theories have the feature that we may be able to observe objects without observing the properties these objects enjoy. The phenomenon of partial observability will be very important for our discussion.

The considerations of universality and partial observability will be reflected in the kinds of axioms that capture the empirical content of a theory. Recall GARP, which states that the revealed preference pair $\langle \succeq^R, \succ^R \rangle$ is acyclic. Acyclicity of a revealed preference pair is a universal theory, but it is more. Using the universal quantifier $\forall$ and the negation symbol $\neg$, we can write acyclicity succinctly as for all n,

$$\forall x_1 ... \forall x_n \neg \left(\bigwedge_{i=1}^{n-1} (x_i \succeq^R x_{i+1}) \wedge (x_n \succ^R x_1) \right). \tag{13.1}$$

The structure of Equation (13.1) explains its falsifiability: it begins with universal quantification, followed by a negation, followed by a conjunction $\wedge$ (meaning "and"), followed by basic properties about observables. Specifically, such "basic" properties amount to *observable* relations among observables. This mathematical equation is therefore a Universal Negation of Conjunction of Atomic Formulas: it is an *UNCAF* formula. Negating atomic formulas means that one rules out that a particular (observable) relation holds among observable entities or quantities.

Contrast GARP with the hypothesis that $\succeq^R$ is a preference relation (a weak order) and $\succ^R$ is its strict part. This hypothesis has a universal axiomatization, but *not* of the UNCAF form. We shall see that this distinction matters. The hypothesis that $\succeq^R$ is a weak order and $\succ^R$ is its strict part has the following axiomatization:

I) $\forall x \forall y (x \succeq^R y) \vee (y \succeq^R x)$
II) $\forall x \forall y \forall z (x \succeq^R y) \wedge (y \succeq^R z) \rightarrow (x \succeq^R z)$
III) $\forall x \forall y (x \succ^R y) \leftrightarrow (x \succeq^R y) \wedge \neg (y \succeq^R x)$.

[1] If one could observe comparisons between all pairs of alternatives, then concavity would clearly be testable.

Some claims made by this theory are falsifiable. For example if we observe that $x \succ^R y$, $y \succ^R z$, and $z \succ^R x$ (a violation of transitivity), then obviously we have falsified the theory. But not *all* claims made by the theory are falsifiable. A pair x, y could be observed without observing any relation between them for example: this is common in the consumption datasets analyzed in Chapters 3–5. Therefore, in the presence of partial observability, completeness cannot be falsified.[2] Interestingly, GARP is usually understood as forming the empirical content of axioms (I), (II), and (III). We demonstrate below the formal sense in which this is true.

13.1 A MODEL FOR OBSERVABLES, DATA, AND THEORIES

There are three important concepts we need to explain. The first is the primitive of the model: the things we can observe are the primitive, and these things will be specified through a *language*. The second is what we mean by a dataset: datasets are finite and consist of partial observations. The third is our notion of a theory: a theory is a formal way of hypothesizing that certain relationships hold between objects of interest.

A first-order *language* $\mathcal{L}$ is given by a finite set of *relation symbols* and, for each relation symbol R, a positive integer n_R, the *arity of* R.

Let $\mathcal{L}$ be a language. An $\mathcal{L}$*-dataset* $\mathcal{D}$ is given by:

I) A finite non-empty set D (the *domain* of $\mathcal{D}$).
II) An n-ary relation $R^{\mathcal{D}}$ over D for every n-ary relation symbol R of $\mathcal{L}$.

Each element $(x_1^*, \ldots, x_{n_R}^*) \in R^{\mathcal{D}}$ is intended to represent the *observation* that $(x_1^*, \ldots, x_n^*)$ stand in relation R. The notion of a dataset is intended to capture the idea that there can only be a finite number of observations.

As an example, consider the language $\mathcal{L}$ that has two binary relation symbols, $\succeq$ and $\succ$. We mean the former to signify revealed weak preference observations, and the latter to signify revealed strict preference observations. Consider a dataset $\mathcal{D}^1$, with domain $D^1 = \{a, b, c\}$, and where we observe that a is revealed weakly preferred to b, and b to c, but we do not observe any strict comparisons. In symbols, $\mathcal{D}^1 = (D^1, \succeq^{\mathcal{D}^1}, \succ^{\mathcal{D}^1})$; $\succeq^{\mathcal{D}^1}$ is the relation given by $a \succeq^{\mathcal{D}^1} b$ and $b \succeq^{\mathcal{D}^1} c$, while $\succ^{\mathcal{D}^1}$ is empty. The example illustrates partial observability: We might theorize that $a \succeq^{\mathcal{D}^1} b$ implies either $b \succeq^{\mathcal{D}^1} a$ or $a \succ^{\mathcal{D}^1} b$, but often data will not contain this kind of information. In fact partial observability is very prominent in economics, for example in the consumption datasets used by the papers described in Chapter 5.

[2] In an environment of full observability (meaning, not partial) and strong rationalization, Eliaz and Ok (2006) investigate choice functions based on maximization of a relation that retains transitivity, but which need not be complete. In such a context, they show that completeness adds real content. They also show how indifference can often be distinguished from incompleteness in such an environment.

We will now define a notion of theory. We are interested in investigating a notion of empirical content, so from this perspective, we imagine that all of the relevant aspects of a theory can be captured by describing the relations among potential observables that we hypothesize hold. To this end, define an *$\mathcal{L}$-structure* $\mathcal{M}$ to consist of the following objects:

I) A nonempty set M (the *domain* of $\mathcal{M}$).
II) An n-ary relation $R^{\mathcal{M}}$ over M for every n-ary relation symbol R of $\mathcal{L}$.

A structure forms a hypothesis about the relations which we might expect to observe, but we never expect to see the entire structure. Rather, we imagine that a dataset is consistent with a structure when all of the observations are members of the structure.

With this in mind, we say that an $\mathcal{L}$-structure $\mathcal{M}$ *rationalizes* an $\mathcal{L}$-dataset $\mathcal{D}$ if the following conditions are satisfied:

I) $D \subseteq M$, where D and M are the domains of $\mathcal{D}$ and $\mathcal{M}$.
II) $R^{\mathcal{D}} \subseteq R^{\mathcal{M}}$.

The definition of rationalization requires that $R^{\mathcal{D}} \subseteq R^{\mathcal{M}}$ rather than that $R^{\mathcal{D}}$ is the restriction of $R^{\mathcal{M}}$ to D. The idea is again simply that we do not imagine that all existing relations are necessarily observed. This is the nature of partial observability: observing only a weak preference for coffee over tea does not refute the possibility that coffee is strictly preferred to tea.

We say that two structures are *isomorphic* if we can relabel the objects across the two structures so that all relations are maintained. Let $\mathcal{M}$ and $\mathcal{N}$ be $\mathcal{L}$-structures with domains M and N respectively. Formally, an *isomorphism from $\mathcal{M}$ to $\mathcal{N}$* is a bijective map $\eta : M \to N$ that preserves the interpretations of all symbols of $\mathcal{L}$: $(a_1, ..., a_{n_R}) \in R^{\mathcal{M}}$ iff $(\eta(a_1), ..., \eta(a_{n_R})) \in R^{\mathcal{N}}$ for every relation symbol R of $\mathcal{L}$ and $a_1, ..., a_{n_R} \in M$.

Informally, two structures are isomorphic if there is no way to use our language to distinguish between them.

Finally, we define a theory to be a class of structures. Formally, an *$\mathcal{L}$-theory T* is a class of structures that is closed under isomorphism. A dataset $\mathcal{D}$ is *T-rationalizable* if there is a structure in T that rationalizes $\mathcal{D}$. Otherwise, $\mathcal{D}$ *falsifies T*.

As an example, consider again the language with two binary symbols, $\succeq$ and $\succ$. We have the theory T_{wo} consisting of the class of triples $(M, \succeq^M, \succ^M)$ such that $\succeq^M$ is a preference relation on M (a weak order), and $\succ^M$ is the strict preference derived from $\succeq^M$. The theory T_{wo} can be thought of as the theory of rational choice. Note that T_{wo} rationalizes the dataset $\mathcal{D}^1$ described above: $\mathcal{D}^1$ has observed objects $D^1 = \{a, b, c\}$, where a is revealed weakly preferred to b, and b to c. The data $\mathcal{D}^1$ is T_{wo}-rationalizable because it can be rationalized, for example, by the set M of all letters in the English alphabet, with $\succeq^M$ being the lexicographic order on M. Of course, there are many other structures in T_{wo} that could rationalize $\mathcal{D}^1$.

We emphasized that $\mathcal{D}^1$ is silent on some aspects of the relationship between a, b, and c; these aspects are not observed, but we do not view this partial observability as a conflict with T_{wo}. Some "theoretically true" relations are simply not observed in the dataset. The structure of all the letters in the English alphabet is a possible rationalization of $\mathcal{D}^1$ in T_{wo}. In this structure a is strictly preferred to c, a property that has not been observed in $\mathcal{D}^1$.

In contrast with data $\mathcal{D}^1$, consider the dataset $\mathcal{D}^2$; where $D^2 = \{a,b,c\}$ (the same as D^1), but where we observe no weak comparisons, and instead observe that

$$a \succ^{\mathcal{D}^2} b \succ^{\mathcal{D}^2} c \succ^{\mathcal{D}^2} a.$$

No structure in T_{wo} could rationalize $\mathcal{D}^2$ because the "theoretical" strict preference $\succ^M$ in such a structure would need to exhibit the cyclic comparisons $a \succ^M b \succ^M c \succ^M a$. This is impossible for a weak order.

Another example of a theory is the theory of utility maximization, denoted T_u, which is the set of triples $(M, \succeq^M, \succ^M)$ in T_{wo} such that there is $u : M \to \mathbf{R}$ with $x \succeq^M y$ iff $u(x) \geq u(y)$. Note that

$$T_u \subsetneq T_{wo}.$$

The theory of utility maximization is more stringent than T_{wo}. But T_u can also rationalize $\mathcal{D}^1$ (but not $\mathcal{D}^2$). In fact, it is easy to see that any dataset that is T_{wo}-rationalizable is also T_u-rationalizable, even though utility maximization is more stringent than rational choice. Put differently, one can weaken T_u to T_{wo} *without observable consequences*. When that happens we shall say that the two theories have the same empirical content.

13.1.1 Empirical content

We want the empirical content of a theory to capture all of the observable consequences of that theory, but no other consequences. To this end, we want to weaken the theory as much as possible without changing the observable consequences of the theory, removing all non-observable consequences. For example, we can obtain a new theory by adding structures to T_u (thus weakening T_u) without changing the datasets that falsify the new theory. The empirical content of T_u is the most one can weaken T_u in this fashion.

Hence we define the *empirical content* of a theory T, denoted $\mathrm{ec}(T)$, to be the class of all structures $\mathcal{M}$ that do not rationalize any dataset that falsifies T. The main result of this chapter is that the empirical content of a theory is captured by the UNCAF axioms that are true in that theory.

Given a language $\mathcal{L}$, we can write formulas using the symbols in $\mathcal{L}$. In addition to the relation symbols specified by $\mathcal{L}$, we shall use standard logical symbols: the *quantifiers* "exists" ($\exists$) and "for all" ($\forall$); "not" ($\neg$); the *logical connectives* "and" ($\wedge$) and "or" ($\vee$); a countable set of *variable symbols* $x, y, z, u, v, w, \ldots$; parentheses "(" and ")"; and equality and inequality symbols "$=$" and "$\neq$".

Strings of symbols are put together to form *axioms*. Rules for the formation of axioms can be found, for example, in Marker (2002).

We are primarily concerned with a special class of axioms, the UNCAF axioms. These are defined as follows. First we must define the notion of an *atomic formula*.

An *atomic formula* φ of a language $\mathcal{L}$ consists of either

I) $t_1 = t_2$ or $t_1 \neq t_2$, where t_1, t_2 are variable symbols or
II) $R(t_1, \ldots, t_{n_R})$ where R is a relation symbol of $\mathcal{L}$ and $t_1, \ldots, t_{n_R}$ are variable symbols.

Atomic formulas are closely related to the notion of observation discussed above. Let us consider again the language with two binary relation symbols, $\succeq$ and $\succ$. In this example, all atomic formulas use at most two variable symbols. For example, the string $x \succeq y$ is an atomic formula, as is $x \succ y$. These strings are unquantified and as such do not yet form axioms. But one can imagine that an observation might consist of some pair a and b for which a is observed to stand in relation $\succeq$ to b.

We will form axioms out of the atomic formulas. The axioms are intended to be statements precluding the existence of certain finite collections of observations from holding. To this end, let $\mathcal{L}$ be a language. A *universal negation of a conjunction of atomic formulas (UNCAF)* axiom is a string of the form

$$\forall v_1 \forall v_2 \ldots \forall v_n \neg\, (\varphi_1 \wedge \varphi_2 \cdots \wedge \varphi_m)$$

where $\varphi_1, \varphi_2, \ldots, \varphi_m$ are atomic formulas with variables from $v_1, \ldots, v_n$.

As an example, consider again the language that has two binary relation symbols, $\succeq$ and $\succ$. An example of an UNCAF axiom in this language is:

$$\forall x \forall y \neg\, (x \succeq y \wedge y \succ x)\,;$$

which we might call the weak axiom of revealed preference (WARP). Similarly, GARP can be seen to be an UNCAF axiom. In a different language, congruence (recall Chapter 2) is UNCAF.

Let Γ be a set of UNCAF axioms of $\mathcal{L}$. Let $\mathcal{T}(\Gamma)$ consist of the structures for which all axioms in Γ are true; thus, $\mathcal{T}(\Gamma)$ is a theory. If $T = \mathcal{T}(\Gamma)$ for some set Γ of axioms, we say that Γ is an UNCAF *axiomatization* of T.

Given a theory T, let $\mathrm{uncaf}(T)$ be the set of UNCAF axioms that are true in all members of T. The following result establishes that the empirical content of a theory always has an UNCAF axiomatization. This is true whether or not the theory itself does. Moreover, one such UNCAF axiomatization consists of the UNCAF axioms true for every structure in the theory.

Theorem 13.1 *For every theory T, $ec(T)$ is the theory axiomatized by the UNCAF axioms that are true in T: $ec(T) = \mathcal{T}(\mathrm{uncaf}(T))$.*

Thus, in the presence of partial observability, UNCAF axioms, not universal ones, properly describe the empirical content of the model. In the context of the examples we have been using, Theorem 13.1 presents the formal sense in which GARP is the empirical content of completeness and transitivity.

Theories that coincide with their empirical content are in some sense special. They cannot be weakened in any way without adding falsifying datasets. We shall see in 13.2 an example of a theory with $T = \mathrm{ec}(T)$.

13.1.2 Relative theories

In economics, a researcher often wants to take certain hypotheses as being given. For example, economic theorists often view continuity axioms as a technical assumption. By itself, continuity has no empirical content. Hence, they are not interested in testing for this property, though it is often useful for providing a representation. Even though continuity by itself usually has no empirical content, the axiom may have empirical content when imposed with other axioms. So what we really care about are the empirical implications of a preference in the presence of the hypothesized continuity.

Moreover, there are often obvious constraints imposed on us by the structure of the model. We can imagine an individual with preferences over bundles of coffee and tea. Bundles of coffee and tea are elements in a linear space. It would not be interesting to "test" the axioms for a vector space. One can then talk about the empirical content of the economically meaningful theory, relative to the theory of linear spaces.

Consider two theories, T and T', where $T \subseteq T'$. We can define the *empirical content of T relative to T'*, written $\mathrm{ec}_{T'}(T)$, as the class of all structures $\mathcal{M} \in T'$ that do not rationalize any dataset that falsifies T, i.e.,

$$\mathrm{ec}_{T'}(T) = \mathrm{ec}(T) \cap T'. \tag{13.2}$$

The following is an immediate consequence of Theorem 13.1.

Corollary 13.2 *For any theories T and T' such that $T \subseteq T'$, $ec_{T'}(T) = \mathcal{T}(\mathrm{uncaf}(T)) \cap T'$.*

We say that a collection of UNCAF axioms Λ is an UNCAF axiomatization of T relative to T' if $T = \mathcal{T}(\Lambda) \cap T'$. Corollary 13.2 implies that the empirical content of T relative to T' admits an UNCAF axiomatization relative to T'.

13.2 APPLICATION: *STATUS QUO* PREFERENCES

As mentioned above, theories that satisfy $T = \mathrm{ec}(T)$ are particularly interesting. As an application, we discuss a recent theory of choice in the presence of *status quo* due to Masatlioglu and Ok. There is a sense in which their theory makes no non-falsifiable claims.

Let $\mathcal{L}$ be a language including the binary relation $\in$ and the ternary relations $c, \tilde{c}$. The latter two are meant to express "chosen" and "not chosen." We define the theory of choice with *status quo* T_{csq} to be the class of structures $(M, \in^{\mathcal{M}},$

$c^{\mathcal{M}}, \tilde{c}^{\mathcal{M}}$) whereby there is some set X, a collection $\Sigma \subseteq 2^X \backslash \{\varnothing\} \times X$ satisfying $(E,b) \in \Sigma$ implies $b \in E$, and a nonempty valued function $c^* : \Sigma \rightarrow 2^M \backslash \{\varnothing\}$ for which $c^*(E,b) \subseteq E$ for all $(E,b) \in \Sigma$ for which the following are satisfied:

I) $M = X \cup \Sigma$
II) $\in^{\mathcal{M}}$ is the usual set theoretic relation
III) $c^{\mathcal{M}}(a,b,E)$ if and only if $a \in c^*(E,b)$
IV) $\tilde{c}^{\mathcal{M}}(a,b,E)$ if and only if $a \notin c^*(E,b)$;

as well as all structures isomorphic to these. The idea here is that each budget set E possesses a *status quo* b. We observe the choices made (and not made) from budget sets.

The theory of *status quo* rationalizable choice is the subtheory $T_{sq} \subseteq T_{csq}$ whereby there exists a function $Q : X \rightarrow 2^X \backslash \{\varnothing\}$ such that for all $x \in X$, $x \in Q(x)$, and a complete and transitive binary relation $\succeq$ for which the corresponding function c^* can be expressed as $c^*(E,b) = \arg\max_{\succeq} Q(b) \cap E$. The idea is that the *status quo* alternative determines a "reference set" from which the agent will choose.[3]

Theorem 13.3 $ec_{T_{csq}}(T_{sq}) = T_{csq}$.

Proof. We provide an explicit syntactic characterization. The formula

$$(x \in E) \wedge (y \in E) \wedge c(x,d,E)$$

is abbreviated $x \succeq (E)_d\, y$, and the formula

$$(x \in E) \wedge (y \in E) \wedge c(x,d,E) \wedge \tilde{c}(y,d,E)$$

is abbreviated $x \succ (E)_d\, y$. Note that each of these formulas is a conjunction of atomic formulas.

Similarly, we define, for a structure $(M, \in^{\mathcal{M}}, c^{\mathcal{M}}, \tilde{c}^{\mathcal{M}}) \in T_{csq}$, the ternary relations $x \succeq_d^{\mathcal{M}} y$ to mean that there is E for which $x, y \in E$ and $x \in c^*(d,E)$, and $x \succ_d^{\mathcal{M}} y$ means there is E for which $x, y \in E$ and $x \in c^*(d,E)$ and $y \notin c^*(d,E)$ (here, we suppress the dependence of the relation on E as it is not needed).

Now, a *cycle* for a structure $\mathcal{M} \in T_{csq}$ is a collection $x_1, \ldots, x_n$, $y_1, \ldots, y_n$, and $z_1, \ldots, z_n$ all in M such that $x_{i+1} \neq y_i$ and $x_i \succeq_{z_i}^{\mathcal{M}} x_{i+1}$ with at least one strict part, and for each z_i, $x_{i+1} \succeq_{z_i}^{\mathcal{M}} y_i$ *or* $x_{i+1} = z_i$.

Note that the collection of axioms which rule out all cycles is UNCAF, as each formula $x \succeq (E)_d\, y$ is a conjunction of atomic formulas, as is the formula $x \succ (E)_d\, y$. In other words, the UNCAF list of axioms given by

$$\forall x_1 \ldots \forall z_n \forall E_1 \ldots \forall E_n \forall F_1 \ldots \forall F_n \neg \left(\bigwedge_{i=1}^{n} (x_i R_i (E_i)_{z_i} x_{i+1}) \wedge (Q_i(x_{i+1}, y_i, z_i, F_i)) \right),$$

[3] Masatlioglu and Ok (2014) also allow there to be no *status quo*, which they represent with a *status quo* "alternative" of $\lozenge$. We could accommodate this easily by introducing a constant symbol for $\lozenge$, and none of the results would change. However, the analysis would become notationally much more burdensome.

where each R_i is either $\succ$ or $\succeq$ and at least one is $\succ$, and each $Q_i(x_{i+1}, y_i, z_i, F_i)$ is either the expression $x_{i+1} \succeq (F_i)_{z_i}\ y_i$ or the expression $x_{i+1} = z_i$, is the appropriate collection of sentences.

Finally, given $\mathcal{M} \in T_{csq}$, we have its associated ternary relations $\succeq^{\mathcal{M}}$ and $\succ^{\mathcal{M}}$. We claim that $\mathcal{M} \in T_{sq}$ if and only if it has no cycles.

To this end, suppose that $\mathcal{M} \in T_{sq}$. Then $\mathcal{M}$ is isomorphic to a triple (X, Σ, c^*). There is a binary relation R and a function Q as stated in the definition. Suppose for a contradiction that there is a cycle. Each $x_{i+1} \in Q(z_i)$, since $x_{i+1} \succeq^{\mathcal{M}}_{z_i} y_i$ (implying it is chosen at some point when z_i is the *status quo*) or $x_{i+1} = z_i$, which again implies that $x_{i+1} \in Q(z_i)$. Therefore, $x_i \succeq^{\mathcal{M}}_{z_i} x_{i+1}$ implies $x_i\, R\, x_{i+1}$ and $x_i \succ^{\mathcal{M}}_{z_i} x_{i+1}$ implies $x_i\, R\, x_{i+1}$ but not $x_{i+1}\, R\, x_i$, contradicting transitivity.

Conversely, suppose $\mathcal{M} \in T_{csq}$ and that there are no cycles. Without loss of generality, we may assume that $\mathcal{M}$ is specified by a triple (X, Σ, c^*). We define $Q(d) = \{y : \exists E \in \Sigma \text{ such that } y \in c^*(d, E)\}$. Define $\overline{\succeq}_d = \{(x, y) \in \succeq_d \colon y \in Q(d)\}$ and $\overline{\succ}_d = \{(x, y) \in \succ_d \colon y \in Q(d)\}$. Finally, define $\overline{\succeq} = \bigcup_{d \in X} \overline{\succeq}_d$ and $\overline{\succ} = \bigcup_{d \in X} \overline{\succeq}_d$. Then because there are no cycles, there exist no $x_1, \ldots, x_n$ for which $x_1 \overline{\succeq} \ldots \overline{\succeq} x_n$, where at least one $\overline{\succeq}$ is $\overline{\succ}$. By Theorem 1.5, there is a complete and transitive R for which $x \overline{\succeq} y$ implies $x\, R\, y$ and $x \overline{\succ} y$ implies $x\, R\, y$. The pair Q, R then *status quo* rationalizes c^*.

13.3 CHOICE THEORY AND EMPIRICAL CONTENT

As mentioned earlier in the chapter, theories that coincide with their empirical content are in some sense special. We shall here consider the structure of such theories relative to a theory of choice.

The theory of rationalizable choice functions enjoys a very interesting property. Recall Chapter 2 and, in particular, Theorem 2.6. The theorem characterizes the empirical content of a preference relation by the congruence axiom. Congruence is a collection of first-order axioms precluding the existence of certain types of revealed preference cycles. Importantly, congruence can be described in a very parsimonious language: the language is able to express the relations $\in$, $\notin$, as well as the properties of being chosen or not. This means that the data the economist must possess in order to falsify the model are quite simple.

In this section, we demonstrate a very simple result claiming that most choice theories do not share this property. As motivation, consider the following simple example.

Example 13.4 *An economist asks whether a given individual maximizes a preference relation (a weak order). She observes three budget sets, A, B, and D, and three potential choices, x, y, and z. She sees that each of x and y are feasible in A, y and z are feasible in B, and x and z are feasible in D. She also sees that x is chosen from A, y is chosen from B, and z is chosen from D, while x* *is never chosen from D. These data clearly refute the hypothesis of preference maximization.*

To establish this refutation, the economist did not see every feasible option from each of the three budget sets or any "global set" on which the choice function might potentially be defined, nor did she need to see what the individual would choose from an unrelated budget E.

Consider two languages. The first, $\mathcal{L} = \{\in, \notin, c, \tilde{c}\}$, includes four binary predicates. The predicates $\in$ and $\notin$ are to be understood in their usual way, while $c(a,B)$ means a is chosen from B, and $\tilde{c}(a,B)$ means a is not chosen from B. The second language is more expressive: $\mathcal{L}' = \{\in, \notin, c, \tilde{c}, \subseteq, \nsubseteq\}$. The binary relations $\subseteq$ and $\nsubseteq$ are given their usual interpretation.

A choice function consists of a triple (X, Σ, c), where X is a set, $\Sigma \subseteq 2^X \backslash \{\varnothing\}$, and $c : \Sigma \to 2^X \backslash \{\varnothing\}$, so that for all $E \in \Sigma$, $c(E) \subseteq E$. Each choice function is naturally identified with a structure in either language.

In either language, $\mathcal{L}$ or $\mathcal{L}'$, a choice theory T is a class of choice functions and all structures isomorphic to an element of this class. The choice theory which consists of all choice functions is written T_{choice} in language $\mathcal{L}$ and T'_{choice} in language $\mathcal{L}'$, respectively.

Say that the $\mathcal{L}$-theory T is *rich* if for all (X, Σ, c) in T, there is $z \notin X$ and a choice function $(X \cup \{z\}, \Sigma', c') \in T$, where $\Sigma = \{E \cap X : E \in \Sigma'\}$ and for all $E \in \Sigma'$, $c'(E) = c(E \cap X)$. Intuitively, a rich choice theory is one with the property that, for any of its structures, we can add an alternative that is never chosen to the domain. Say that a choice theory satisfies condition α if all choice functions in the theory satisfy condition α.

When a theory satisfies $\mathrm{ec}(T) \cap T_{choice} = T$, then we say that it makes no non-falsifiable claims relative to the theory of choice. We are now in a position to show that, under some conditions, the property of not making non-falsifiable claims can imply WARP.

Theorem 13.5 *Suppose that T is rich, satisfies condition α, and that $ec(T) \cap T_{choice} = T$. Then every $(X, \Sigma, c) \in T$ satisfies WARP. Further, the class of choice functions satisfying WARP is the maximal T which is rich, satisfies condition α, and $ec(T) \cap T_{choice} = T$.*

Proof. Suppose by way of contradiction that $(X, \Sigma, c) \in T$ violates WARP. Thus, there are $x, y \in X$ and $E, F \in \Sigma$ for which $x, y \in E \cap F$, $x \in c(E)$, $y \in c(F)$, and $x \notin c(F)$. Appeal to richness and consider $z \notin X$. Let Σ^* consist of all budgets of Σ, with the exception that z has been added to E. The choice function c^* then coincides with c, and $c^*(E \cup \{z\}) = c(E)$.

Now consider the following choice function: $X' = \{x, y, z\}$, $\Sigma' = \{E' = \{x, y, z\}, F' = \{x, y\}\}$, and $c'(\{x, y, z\}) = c(E) \cap \{x, y, z\}$, $c'(\{x, y\}) = \{y\}$. This choice function is clearly not a member of T as it violates condition α: $x \in c'(\{x, y, z\})$, but $x \notin c'(\{x, y\})$. However, every dataset consistent with (X', Σ', c') can be rationalized by $(X \cup \{z\}, \Sigma^*, c^*)$. This is a contradiction.

Clearly, the class of choice functions satisfying WARP has the desired property (WARP is an UNCAF axiom, so the result follows from Corollary 13.2).

A special example of choice theories are choice theories rationalizable by some binary relation. Let $\mathcal{R}$ be a class of complete binary relations. We define the theory of $\mathcal{R}$-rationalizable choice, $T_{\mathcal{R}}$, to be the class of choice functions (X,Σ,c) for which there exists $R \in \mathcal{R}$ such that for all $E \in \Sigma$, $c(E) = \{x \in E : x\,R\,y \text{ for all } y \in E\}$.

Let the relation $\succeq^*$ on $\mathbf{Z}_+ \cup \{\omega\}$ be defined so that for all $n,m \in \mathbf{Z}_+$, we have $n \succeq^* m$ if and only if $n \geq m$, and otherwise, $\omega \sim^* n$ for all $n \in \mathbf{Z}_+$.

A simple corollary follows.

Corollary 13.6 *For the following $\mathcal{R}$, $ec(T_{\mathcal{R}}) \cap T_{choice} \neq T_{\mathcal{R}}$:*

- *$\mathcal{R}$ is the set of all complete binary relations.*
- *$\mathcal{R}$ is the set of all quasitransitive and complete binary relations.*
- *$\mathcal{R}$ is the set of all complete binary relations for which there exists a pair of linear orders $\succeq_1$ and $\succeq_2$ for which $\succ = \succ_1 \cap \succ_2$. These are the set of 2-Pareto rationalizable choice functions.*

Proposition 13.7 *If $\succeq^* \in \mathcal{R}$, then $ec(T_{\mathcal{R}}) \cap T'_{choice} \neq ec(T_{\mathcal{R}})$.*

Proof. Let $X = \mathbf{Z}_+ \cup \{\omega\}$ and let us consider the collection Σ which includes all binary subsets, as well as the set X itself. The choice function is specified by $c(X) = \{\omega\}$, $c(\{i,\omega\}) = \{i,\omega\}$ for all $i \in \mathbf{Z}_+$, and finally, $c(\{i,j\}) = \{j\}$ when $j > i$. Then $(X,\Sigma,c) \in T_{\mathcal{R}}$, and has the feature that ω is uniquely chosen from X, but it beats no $i \in \mathbf{Z}_+$.

Consider the following choice function: (X',Σ',c'), where $X' = \{1,2,\omega\}$, $\Sigma' = \{\{1,2\},\{2,\omega\},\{1,2,\omega\}\}$, and $c'(\{1,2\}) = \{2\}$, $c'(\{2,\omega\}) = \{2,\omega\}$, and $c'(\{1,2,\omega\}) = \{\omega\}$. Clearly $(X',\Sigma',c') \notin T_{\mathcal{R}}$ as if it were, any R which rationalizes c' must have that $2\;R\;1$ and $2\;R\;\omega$, which would imply that $2 \in c'(\{1,2,\omega\})$, a contradiction.

However any dataset contained in (X',Σ',c') can be rationalized by (X,Σ,c), a contradiction.

Proposition 13.7 demonstrates that none of the choice theories mentioned in Corollary 13.6 are equivalent to their empirical content, even when we can express set containment.

13.4 CHAPTER REFERENCES

Theorem 13.1 and Corollary 13.2 are due to Chambers, Echenique, and Shmaya (2014). This result is related to a theorem of Tarski (1954), which characterizes those theories that are universally axiomatizable. Tarski's result relies on the axiom of choice, while Theorem 13.1 does not. The working paper version of Chambers, Echenique, and Shmaya (2014) has results for languages with constants and function symbols. A collection of papers apply Tarski's result to the issue of falsifiability; see Simon and Groen (1973); Simon (1979, 1983); Rynasiewicz (1983); Simon (1985); Shen and Simon (1993).

Popper (1959) is a seminal reference in the philosophy of science, viewing falsifiable theories as those that admit universal axiomatizations. Much early literature in philosophy of science was concerned with whether the restrictions on observable relations imposed by axioms involving unobservable relations could be expressed in terms of observable relations alone. Craig (1956) provides a seminal result in this direction.

Adams, Fagot, and Robinson (1970) seem to be the first social scientists to discuss empirical content in a formal sense (see also Pfanzagl, Baumann, and Huber, 1971, and Adams, 1992). This work defines two theories to be empirically equivalent if the set of all axioms (of a certain type) consistent with one theory is equivalent to the set of all axioms consistent with the other. These works do not provide a general characterization of the axiomatic structure of empirical content, but rather focus on characterizing the empirical content of specific theories. Finally, there is an approach in philosophy of science called *structuralism*, which also comes close to the approach we have taken here. These works also adopt a model-theoretic perspective to investigating theories. Sneed (1971) is a classic reference as applied to physics. Stegmüller, Balzer, and Spohn (1982) present a collection applications of these ideas to economics.

Schipper (2009) has a notion of theory that is similar to ours.

The theory discussed in Section 13.2 is due to Masatlioglu and Ok (2014). The analysis of this section appeared in a working version of Chambers, Echenique, and Shmaya (2014).

References

Adams, E. (1992). “On the Empirical Status of Measurement Axioms: The Case of Subjective Probability,” in *Philosophical and Foundational Issues in Measurement Theory*, edited by C. W. Savage and P. Erlich, pp. 53–73. Hillsdale, NJ: Lawrence Erlbaum Associates.

Adams, E., R. Fagot, and R. Robinson (1970). “On the Empirical Status of Axioms in Theories of Fundamental Measurement,” *Journal of Mathematical Psychology*, 7(3), 379–409.

Afriat, S. N. (1967). “The Construction of Utility Functions from Expenditure Data,” *International Economic Review*, 8(1), 67–77.

——— (1972). “Efficiency Estimation of Production Functions,” *International Economic Review*, 13(3), 568–598.

Agranov, M., and P. Ortoleva (2013). “Stochastic Choice and Hedging,” Discussion paper, California Institute of Technology.

Ahn, D., S. Choi, D. Gale, and S. Kariv (2014). “Estimating Ambiguity Aversion in a Portfolio Choice Experiment,” *Quantitative Economics*, 5(2), 195–223.

Aleskerov, F., D. Bouyssou, and B. Monjardet (2007). *Utility Maximization, Choice and Preference*. Berlin *et al.*: Springer-Verlag.

Allais, M. (1953). “Le Comportement de l’Homme Rationnel Devant le Risque: Critique des Postulats et Axiomes de l’École Américaine,” *Econometrica*, 21(4), 503–546.

Ambrus, A., and K. Rozen (2014). “Rationalising Choice with Multi-Self Models,” *The Economic Journal*, forthcoming.

Andreoni, J., B. J. Gillen, and W. T. Harbaugh (2013). “Power Indices for Revealed Preference Tests,” working paper, UC San Diego.

Andreoni, J., and J. Miller (2002). “Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism,” *Econometrica*, 70(2), 737–753.

Anscombe, F. J., and R. J. Aumann (1963). “A Definition of Subjective Probability,” *Annals of Mathematical Statistics*, 34(1), 199–205.

Arrow, K. (1951). *Social Choice and Individual Values*. New Haven, CT: Yale University Press.

——— (1959). “Rational Choice Functions and Orderings,” *Economica*, 26(102), 121–127.

Asplund, E. (1970). “A Monotone Convergence Theorem for Sequences of Nonlinear Mappings,” in *Proceedings of Symposia in Pure Mathematics*, Vol. 18, pp. 1–9.

Azrieli, Y. (2011). “Axioms for Euclidean Preferences with a Valence Dimension,” *Journal of Mathematical Economics*, 47(4–5), 545–553.

Azrieli, Y., C. P. Chambers, and P. J. Healy (2012). “Incentives in Experiments: A Theoretical Analysis,” working paper.

Ballester, M. A., and G. Haeringer (2011). “A Characterization of the Single-Peaked Domain,” *Social Choice and Welfare*, 36(2), 305–322.

Bandyopadhyay, T., I. Dasgupta, and P. Pattanaik (1999). “Stochastic Revealed Preference and the Theory of Demand,” *Journal of Economic Theory*, 84(1), 95–110.

——— (2004). “A General Revealed Preference Theorem for Stochastic Demand Behavior,” *Economic Theory*, 23(3), 589–599.

Banerjee, S., and J. H. Murphy (2006). “A Simplified Test for Preference Rationality of Two-Commodity Choice,” *Experimental Economics*, 9(1), 67–75.

Banker, R., A. Charnes, and W. Cooper (1984). “Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis,” *Management Science*, 30(9), 1078–1092.

Banker, R., and A. Maindiratta (1988). “Nonparametric Analysis of Technical and Allocative Efficiencies in Production,” *Econometrica*, 56(6), 1315–1332.

Bar-Shira, Z. (1992). “Nonparametric Test of the Expected Utility Hypothesis,” *American Journal of Agricultural Economics*, 74(3), 523–533.

Barberá, S., and P. K. Pattanaik (1986). “Falmagne and the Rationalizability of Stochastic Choices in Terms of Random Orderings,” *Econometrica*, 54(3), 707–715.

Bartz, S., H. Bauschke, J. Borwein, S. Reich, and X. Wang (2007). “Fitzpatrick Functions, Cyclic Monotonicity and Rockafellar’s Antiderivative,” *Nonlinear Analysis*, 66(5), 1198–1223.

Basu, K. (1984). “Fuzzy Revealed Preference Theory”, *Journal of Economic Theory*, 32(2), 212–227.

Battalio, R., J. Kagel, W. Winkler, E. Fischer, R. Basmann, and L. Krasner (1973). “A Test of Consumer Demand Theory Using Observations of Individual Consumer Purchases,” *Western Economic Journal*, 11(4), 411–428.

Bayer, R., S. Bose, M. Polisson, L. Renou, and J. Quah (2012). “Ambiguity Revealed,” Mimeo, University of Essex.

Beatty, T. K. M., and I. A. Crawford (2011). “How Demanding Is the Revealed Preference Approach to Demand?” *The American Economic Review*, 101(6), 2782–2795.

Becker, G. M., M. H. DeGroot, and J. Marschak (1964). “Measuring Utility by a Single-Response Sequential Method,” *Behavioral Science*, 9(3), 226–232.

Becker, G. S. (1962). “Irrational Behavior and Economic Theory,” *The Journal of Political Economy*, 70(1), 1–13.

Berge, C. (1963). *Topological Spaces*. Edinburgh and London: Oliver & Boyd.

Berge, C. (2001). *Theory of Graphs*. Mineola, NY: Dover.

Bernheim, B. D., and A. Rangel (2007). “Toward Choice-Theoretic Foundations for Behavioral Welfare Economics,” *The American Economic Review*, 97(2), 464–470.

——— (2009). "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics," *The Quarterly Journal of Economics*, 124(1), 51–104.

Billot, A., and J.-F. Thisse (2005). "Stochastic Rationality and Möbius Inverse," *International Journal of Economic Theory*, 1(3), 211–217.

Birkhoff, G. (1946). "Tres Observaciones Sobre el Álgebra Lineal," *Univ. Nac. Tucumán Rev. Ser. A*, 5, 147–151.

Blackorby, C., W. Bossert, and D. Donaldson (1995). "Multi-Valued Demand and Rational Choice in the Two-Commodity Case," *Economics Letters*, 47(1), 5–10.

Block, H., and J. Marschak (1960). "Random Orderings and Stochastic Theories of Responses," in *Contributions to Probability and Statistics*, edited by I. Olkin, S. Ghurye, W. Hoeffding, W. G. Madow, and H. B. Mann, Vol. 2, pp. 97–132. Palo Alto, CA: Stanford University Press.

Blundell, R. W., M. Browning, and I. A. Crawford (2003). "Nonparametric Engel Curves and Revealed Preference," *Econometrica*, 71(1), 205–240.

——— (2008). "Best Nonparametric Bounds on Demand Responses," *Econometrica*, 76(6), 1227–1262.

Bochnak, J., M. Coste, and M. Roy (1998). *Real Algebraic Geometry*. Berlin *et al.*: Springer-Verlag.

Bogomolnaia, A., and J. Laslier (2007). "Euclidean Preferences," *Journal of Mathematical Economics*, 43(2), 87–98.

Border, K. C. (1992). "Revealed Preference, Stochastic Dominance, and the Expected Utility Hypothesis," *Journal of Economic Theory*, 56(1), 20–42.

Börgers, T. (1993). "Pure Strategy Dominance," *Econometrica*, 61(2), 423–430.

Bossert, W. (1993). "Continuous Choice Functions and the Strong Axiom of Revealed Preference," *Economic Theory*, 3(2), 379–385.

Bossert, W., and Y. Sprumont (2002). "Core Rationalizability in Two-Agent Exchange Economies," *Economic Theory*, 20(4), 777–791.

——— (2013). "Every Choice Function is Backwards-Induction Rationalizable," *Econometrica*, 81(6), 2521–2534.

Bossert, W., Y. Sprumont, and K. Suzumura (2005). "Maximal-Element Rationalizability," *Theory and Decision*, 58(4), 325–350.

Bossert, W., and K. Suzumura (2010). *Consistency, Choice, and Rationality*. Harvard University Press.

——— (2012). "Revealed Preference and Choice Under Uncertainty," *SERIEs: Journal of the Spanish Economic Association*, 3(1), 247–258.

Bronars, S. G. (1987). "The Power of Nonparametric Tests of Preference Maximization," *Econometrica*, 55(3), 693–698.

Brown, D., and F. Kubler (2008). *Computational Aspects of General Equilibrium Theory: Refutable Theories of Value*. New York *et al.*: Springer.

Brown, D. J., and C. Calsamiglia (2007). "The Nonparametric Approach to Applied Welfare Analysis," *Economic Theory*, 31(1), 183–188.

Brown, D. J., and R. L. Matzkin (1996). "Testable Restrictions on the Equilibrium Manifold," *Econometrica*, 64(6), 1249–1262.

Brown, D. J., and C. Shannon (2000). "Uniqueness, Stability, and Comparative Statics in Rationalizable Walrasian Markets," *Econometrica*, 68(6), 1529–1539.

Browning, M. (1989). "A Nonparametric Test of the Life-Cycle Rational Expectations Hypothesis," *International Economic Review*, 30(4), 979–992.

Browning, M., and P. A. Chiappori (1998). "Efficient Intra-Household Allocations: A General Characterization and Empirical Tests," *Econometrica*, 66(6), 1241–1278.

Burghart, D. R., P. W. Glimcher, and S. C. Lazzaro (2013). "An Expected Utility Maximizer Walks into a Bar...," *Journal of Risk and Uncertainty*, 46(3), 215–246.

Cantor, G. (1895). "Beiträge zur Begründung der transfiniten Mengenlehre," *Mathematische Annalen*, 46(4), 481–512.

Carvajal, A., R. Deb, J. Fenske, and J. K.-H. Quah (2013). "Revealed Preference Tests of the Cournot Model," *Econometrica*, 81(6), 2351–2379.

Carvajal, A., and N. González (2014). "On Refutability of the Nash Bargaining Solution," *Journal of Mathematical Economics*, 50, 177–186.

Carvajal, A., I. Ray, and S. Snyder (2004). "Equilibrium Behavior in Markets and Games: Testable Restrictions and Identification," *Journal of Mathematical Economics*, 40(1–2), 1–40, Aggregation, Equilibrium and Observability in Honor of Werner Hildenbrand.

Castillo, M., D. Dickinson, and R. Petrie (2014). "Sleepiness, Choice Consistency, and Risk Preferences," Mimeo, George Mason University.

Chalfant, J. A., and J. M. Alston (1988). "Accounting for Changes in Tastes," *Journal of Political Economy*, 96(2), 391–410.

Chambers, C. P., and F. Echenique (2009a). "Profit Maximization and Supermodular Technology," *Economic Theory*, 40(2), 173–183.

——— (2009b). "Supermodularity and Preferences," *Journal of Economic Theory*, 144(3), 1004–1014.

——— (2014a). "Core Matchings of Markets with Transfers," *AEJ Microeconomics*, forthcoming.

——— (2014b). "On the Consistency of Data with Bargaining Theories," *Theoretical Economics*, 9(1), 137–162.

Chambers, C. P., F. Echenique, and K. Saito (2015). "Testable Implications of Translation Invariance and Homotheticity: Variational, Maxmin, CARA and CRRA Preferences," working paper, California Institute of Technology.

Chambers, C. P., F. Echenique, and E. Shmaya (2010). "On Behavioral Complementarity and its Implications," *Journal of Economic Theory*, 145, 2332–2355.

——— (2011). "Testable Implications of Gross Substitutes in Demand for Two Goods," *American Economic Journal: Microeconomics*, 3(1), 129–136.

——— (2012). "General Revealed Preference Theory," working paper.

——— (2014). "The Axiomatic Structure of Empirical Content," *American Economic Review*, 104(8), 2303–2319.

Chambers, C. P., and T. Hayashi (2012). "Choice and Individual Welfare," *Journal of Economic Theory*, 147(5), 1818–1849.

Chambers, C. P., C. Liu, and S.-K. Martinez (2014). "A Test for Risk-Averse Expected Utility," Mimeo, UC San Diego.

Cherchye, L., I. Crawford, B. De Rock, and F. Vermeulen (2009). "The Revealed Preference Approach to Demand," in *Quantifying Consumer Preferences: Estimating Demand Systems*, edited by D. Slottje, Contributions to Economic Analysis. Bingley, UK: Emerald Group Publishing.

Cherchye, L., T. Demuynck, and B. De Rock (2013). "Nash Bargained Consumption Decisions: A Revealed Preference Analysis," *Economic Journal*, 123(567), 195–235.

——— (2014). "Revealed Preference Analysis for Convex Rationalizations on Nonlinear Budget Sets," *Journal of Economic Theory*, 152, 224–236.

Cherchye, L., T. Demuynck, P. Hjertstrand, and B. De Rock (2014). "Revealed Preference Tests for Weak Separability: An Integer Programming Approach," *Journal of Econometrics*, forthcoming.

Cherchye, L., B. De Rock, and F. Vermeulen (2007). "The Collective Model of Household Consumption: A Nonparametric Characterization," *Econometrica*, 75(2), 553–574.

——— (2009). "Opening the Black Box of Intrahousehold Decision Making: Theory and Nonparametric Empirical Tests of General Collective Consumption Models," *Journal of Political Economy*, 117(6), 1074–1104.

Cherepanov, V., T. Feddersen, and A. Sandroni (2013). "Rationalization," *Theoretical Economics*, 8(3), 775–800.

Chernoff, H. (1954). "Rational Selection of Decision Functions," *Econometrica*, 22(4), 422–443.

Chiappori, P.-A. (1988). "Rational Household Labor Supply," *Econometrica*, 56(1), 63–90.

Chiappori, P.-A., O. Donni, and I. Komunjer (2012). "Learning from a Piece of Pie," *The Review of Economic Studies*, 79(1), 162–195.

Chiappori, P.-A., and J.-C. Rochet (1987). "Revealed Preferences and Differentiable Demand," *Econometrica*, 55(3), 687–691.

Chipman, J. S. (1960). "Stochastic Choice and Subjective Probability," in *Decisions, Values and Groups*, edited by D. Willner, pp. 70–95. Symposium Publications Division, New York: Pergamon Press.

Choi, S., R. Fisman, D. Gale, and S. Kariv (2007). "Consistency and Heterogeneity of Individual Behavior under Uncertainty," *American Economic Review*, 97(5), 1921–1938.

Choi, S., S. Kariv, W. Müller, and D. Silverman (2014). "Who Is (More) Rational?" *American Economic Review*, 104(6), 1518–50.

Chung-Piaw, T., and R. Vohra (2003). "Afriat's Theorem and Negative Cycles," Mimeo, Northwestern University.

Colonius, H. (1984). *Stochastische Theorien individuellen Wahlverhaltens*. Berlin *et al.*: Springer.

Cooper, W. W., L. M. Seiford, and K. Tone (2007). *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, 2nd ed. Berlin *et al.*: Springer.

Coxeter, H. (1969). *Introduction to Geometry*, 2nd ed. New York: John Wiley & Sons, Inc.

Craig, W. (1956). "Replacement of Auxiliary Expressions," *Philosophical Review*, 65(1), 38–55.

Craig, W., and R. L. Vaught (1958). "Finite Axiomatizability Using Additional Predicates," *The Journal of Symbolic Logic*, 23(3), 289–308.

Crawford, I., and B. De Rock (2014). "Empirical Revealed Preference," *Annual Review of Economics*, 6, 503–524.

Davenport, J. H., and J. Heintz (1988). “Real Quantifier Elimination is Doubly Exponential,” *Journal of Symbolic Computation*, 5(1), 29–35.

de Clippel, G., and K. Eliaz (2012). “Reason-Based Choice: A Bargaining Rationale for the Attraction and Compromise Effects,” *Theoretical Economics*, 7(1), 125–162.

de Clippel, G., and K. Rozen (2012). “Bounded Rationality and Limited Datasets,” working paper.

Dean, M., and D. Martin (2013). “Measuring Rationality with the Minimum Cost of Revealed Preference Violations?” Working paper, Brown University.

Deaton, A. S. (1974). “The Analysis of Consumer Demand in the United Kingdom, 1900-1970,” *Econometrica*, 42(2), 341–367.

Deb, R. (1976). “On Constructing Generalized Voting Paradoxes,” *The Review of Economic Studies*, 43(2), 347–351.

Deb, R., and M. M. Pai (2014). “The Geometry of Revealed Preference,” *Journal of Mathematical Economics*, 50, 203–207.

Debreu, G. (1951). “The Coefficient of Resource Utilization,” *Econometrica*, 19(3), 273–292.

Debreu, G. (1954). “Representation of a Preference Ordering by a Numerical Function,” in *Decision Processes*, edited by R. Thrall, C. Coombs, and R. Davis, pp. 159–165. New York: John Wiley & Sons.

——— (1960a). “On An Identity in Arithmetic,” *Proceedings of the American Mathematical Society*, 11(2), 220–221.

——— (1960b). “Review of RD Luce, Individual Choice Behavior: A Theoretical Analysis,” *American Economic Review*, 50(1), 186–188.

——— (1974). “Excess Demand Functions,” *Journal of Mathematical Economics*, 1(1), 15–21.

Degan, A., and A. Merlo (2009). “Do Voters Vote Ideologically?” *Journal of Economic Theory*, 144, 1868–1894.

Diamond, P. A. (1967). “Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparison of Utility: Comment,” *The Journal of Political Economy*, 75(5), 765.

Diewert, W. E. (1973). “Afriat and Revealed Preference Theory,” *The Review of Economic Studies*, 40(3), 419–425.

——— (2012). “Afriat’s Theorem and some Extensions to Choice under Uncertainty*,” *The Economic Journal*, 122(560), 305–331.

Diewert, W. E., and C. Parkan (1983). “Linear Programming Tests of Regularity Conditions for Production Functions,” in *Quantitative Studies on Production and Prices*, edited by W. Eichhorn, R. Henn, K. Neumann, and R. W. Shephard. Berlin *et al.*: Springer Verlag.

Dobell, A. R. (1965). “A Comment on A. Y. C. Koo’s ‘An Empirical Test of Revealed Preference Theory,’ ” *Econometrica*, 33(2), 451–455.

Dowrick, S., and J. Quiggin (1994). “International Comparisons of Living Standards and Tastes: A Revealed-Preference Analysis,” *The American Economic Review*, 84(1), 332–341.

Dushnik, B., and E. W. Miller (1941). “Partially Ordered Sets,” *American Journal of Mathematics*, 63(3), 600–610.

Dvoretzky, A., A. Wald, and J. Wolfowitz (1951). “Elimination of Randomization in Certain Statistical Decision Procedures and Zero-Sum Two-Person Games,” *The Annals of Mathematical Statistics*, 22(1), 1–21.

Dziewulski, P., and J. Quah (2014). “Testing for Production with Complementarities,” Mimeo, University of Oxford.

Echenique, F. (2008). “What Matchings Can Be Stable? The Testable Implications of Matching Theory,” *Mathematics of Operations Research*, 33(3), 757–768.

——— (2013). “Testing for Separability is Hard,” Mimeo, California Institute of Technology.

Echenique, F., T. Imai, and K. Saito (2013). “Testable Implications of Models of Intertemporal Choice: Exponential Discounting and Its Generalizations,” SS working paper 1388, California Instiitute of Technology.

Echenique, F., S. Lee, and M. Shum (2011). “The Money Pump as a Measure of Revealed Preference Violations,” *Journal of Political Economy*, 119(6), 1201–1223.

Echenique, F., S. Lee, M. Shum, and M. B. Yenmez (2013). “The Revealed Preference Theory of Stable and Extremal Stable Matchings,” *Econometrica*, 81(1), 153–171.

Echenique, F., and K. Saito (2013). “Savage in the Market,” *Econometrica*, 83(4), 1467–9.

Ehlers, L., and Y. Sprumont (2008). “Weakened WARP and Top-Cycle Choice Rules,” *Journal of Mathematical Economics*, 44(1), 87–94.

Eliaz, K., and E. A. Ok (2006). “Indifference or Indecisiveness? Choice-Theoretic Foundations of Incomplete Preferences,” *Games and Economic Behavior*, 56(1), 61–86.

Ellsberg, D. (1961). “Risk, Ambiguity, and the Savage Axioms,” *The Quarterly Journal of Economics*, 75(4), 643–669.

Epstein, L. G. (2000). “Are Probabilities Used in Markets?” *Journal of Economic Theory*, 91(1), 86–90.

Epstein, L. G., and A. J. Yatchew (1985). “Non-Parametric Hypothesis Testing Procedures and Applications to Demand Analysis,” *Journal of Econometrics*, 30(1), 149–169.

Falmagne, J. (1978). “A Representation Theorem for Finite Random Scale Systems,” *Journal of Mathematical Psychology*, 18(1), 52–72.

Famulari, M. (1995). “A Household-Based, Nonparametric Test of Demand Theory,” *The Review of Economics and Statistics*, 77(2), 372–382.

——— (2006). “Household Labor Supply and Taxes: A Nonparametric, Revealed Preference Approach,” Mimeo, UC San Diego.

Färe, R., S. Grosskopf, and H. Lee (1990). “A Nonparametric Approach to Expenditure-Constrained Profit Maximization,” *American Journal of Agricultural Economics*, 72(3), 574–581.

Färe, R., S. Grosskopf, and C. K. Lovell (1987). “Nonparametric Disposability Tests,” *Journal of Economics*, 47(1), 77–85.

Farkas, J. (1902). “Theorie der einfachen Ungleichungen,” *Journal für die Reine und Angewandte Mathematik*, 124(124), 1–24.

Farrell, M. (1957). “The Measurement of Productive Efficiency,” *Journal of the Royal Statistical Society. Series A (General)*, 120(3), 253–290.

Fawson, C., and C. R. Shumway (1988). “A Nonparametric Investigation of Agricultural Production Behavior for US Subregions,” *American Journal of Agricultural Economics*, 70(2), 311–317.

Featherstone, A. M., G. A. Moghnieh, and B. K. Goodwin (1995). “Farm-Level Nonparametric Analysis of Cost-Minimization and Profit-Maximization Behavior,” *Agricultural Economics*, 13(2), 109–117.

Ferguson, T. S. (1967). *Mathematical Statistics: A Decision Theoretic Approach*, Vol. 7. New York: Academic Press.

Fiorini, S. (2004). “A Short Proof of a Theorem of Falmagne,” *Journal of Mathematical Psychology*, 48(1), 80–82.

Fishburn, P. C. (1969). “Preferences, Summation, and Social Welfare Functions,” *Management Science*, 16(3), 179–186.

——— (1973a). “Summation Social Choice Functions,” *Econometrica*, 41(6), 1183–1196.

——— (1973b). *The Theory of Social Choice*. Princeton University Press.

——— (1974). “Separation Theorems and Expected Utilities,” *Journal of Economic Theory*, 11, 16–34.

——— (1976). “Representable Choice Functions,” *Econometrica*, 44(5), 1033–1043.

Fitzpatrick, S. (1988). “Representing Monotone Operators by Convex Functions,” in *Workshop/Miniconference on Functional Analysis and Optimization (Canberra 1988), Proceedings of the Centre for Mathematical Analysis, Australian National University*, Vol. 20, pp. 59–65.

Forges, F., and E. Minelli (2009). “Afriat’s Theorem for General Budget Sets,” *Journal of Economic Theory*, 144(1), 135–145.

Fostel, A., H. Scarf, and M. Todd (2004). “Two New Proofs of Afriat‘s Theorem,” *Economic Theory*, 24(1), 211–219.

Galambos, A. (2010). “Revealed Preference in Game Theory,” working paper.

Gale, D. (1960a). “A Note on Revealed Preference,” *Economica*, 27(108), 348–354.

——— (1960b). *The Theory of Linear Economic Models*. University of Chicago press.

Geanakoplos, J. (1984). “Utility Functions for Debreu’s ‘Excess Demands,’ ” *Journal of Mathematical Economics*, 13(1), 1–9.

Ghouila-Houri, A. (1962). “Caractérisation des Graphes Non Orientés dont on Peut Orienter les Arêtes de Manière à Obtenir le Graphe d’une Relation d’Ordre,” *CR Acad. Sci. Paris*, 254, 1370–1371.

Gilboa, I., and D. Schmeidler (1989). “Maxmin Expected Utility with Non-Unique Prior,” *Journal of Mathematical Economics*, 18(2), 141–153.

Gilmore, P., and A. Hoffman (1964). “A Characterization of Comparability Graphs and of Interval Graphs,” *The Canadian Journal of Mathematics*, 16, 539–548.

Gomberg, A. (2011). “Vote Revelation: Empirical Content of Scoring Rules,” in *Political Economy of Institutions, Democracy and Voting*, edited by N. Schofield, and G. Caballero, pp. 411–417. Berlin *et al.*: Springer-Verlag.

——— (2014). “Revealed Votes,” Manuscript.

Green, E., and K. Osband (1991). “A Revealed Preference Theory for Expected Utility,” *The Review of Economic Studies*, 58(4), 677–695.

Green, J., and D. Hojman (2007). “Choice, Rationality, and Welfare Measurement,” working paper.

Green, R. C., and S. Srivastava (1986). “Expected Utility Maximization and Demand Behavior,” *Journal of Economic Theory*, 38(2), 313–323.

Gul, F., P. Natenzon, and W. Pesendorfer (2014). "Random Choice as Behavioral Optimization," *Econometrica*, 82(5), 1873–1912.

Gul, F., and W. Pesendorfer (2006). "Random Expected Utility," *Econometrica*, 74(1), 121–146.

Hanoch, G., and M. Rothschild (1972). "Testing the Assumptions of Production Theory: A Nonparametric Approach," *Journal of Political Economy*, 80(2), 256–275.

Hansson, B. (1968). "Choice Structures and Preference Relations," *Synthese*, 18(4), 443–458.

Harbaugh, W., K. Krause, and T. Berry (2001). "GARP for Kids: On the Development of Rational Choice Behavior," *American Economic Review*, 91(5), 1539–1545.

Hausman, J. A., and D. A. Wise (1978). "A Conditional Probit Model for Qualitative Choice: Discrete Decisions Recognizing Interdependence and Heterogeneous Preferences," *Econometrica*, 46(2), 403–426.

Heufer, J. (2007). "Revealed Preference and the Number of Commodities," working paper.

Hey, J., and N. Pace (2014). "The Explanatory and Predictive Power of Non Two-Stage-Probability Theories of Decision Making Under Ambiguity," *Journal of Risk and Uncertainty*, 49(1), 1–29.

Hicks, J. R. (1956). *A Revision of Demand Theory*. Oxford University Press.

Hoderlein, S. (2011). "How Many Consumers are Rational?" *Journal of Econometrics*, 164(2), 294–309.

Hoderlein, S., and J. Stoye (2014). "Revealed Preferences in a Heterogeneous Population," *Review of Economics and Statistics*, 96(2), 197–213.

Hong, C. S. (1983). "A Generalization of the Quasilinear Mean with Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox," *Econometrica*, 51(4), 1065–1092.

Houthakker, H. S. (1950). "Revealed Preference and the Utility Function," *Economica*, 17(66), 159–174.

——— (1957). "Mr. Newman on Revealed Preference," *Oxford Economic Papers*, 9(2), 234.

Houtman, M., and J. Maks (1985). "Determining All Maximal Data Subsets Consistent with Revealed Preference," *Kwantitatieve Methoden*, 19, 89–104.

Huber, J., J. W. Payne, and C. Puto (1982). "Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis," *Journal of Consumer Research*, 9(1), 90–98.

Hurwicz, L., and M. K. Richter (1979). "Ville Axioms and Consumer Theory," *Econometrica*, 47(3), 603–619.

John, R. (1997). "A Simple Cycle Preserving Extension of a Demand Function," *Journal of Economic Theory*, 72(2), 442–445.

——— (2001). "The Concave Nontransitive Consumer," *Journal of Global Optimization*, 20(3–4), 297–308.

Kalai, G. (2004). "Social Indeterminacy," *Econometrica*, 72(5), 1565–1581.

Kalandrakis, T. (2010). "Rationalizable Voting," *Theoretical Economics*, 5, 93–125.

Kamiya, D. (1963). "A Note on the Strong Axiom of Revealed Preference," *Economica*, 30(117), 83–84.

Kehoe, T., and A. Mas-Colell (1984). “An Observation on Gross Substitutability and the Weak Axiom of Revealed Preference,” *Economics Letters*, 15(3), 241–243.

Kehoe, T. J. (1992). “Gross Substitutability and the Weak Axiom of Revealed Preference,” *Journal of Mathematical Economics*, 21(1), 37–50.

Kihlstrom, R., A. Mas-Colell, and H. Sonnenschein (1976). “The Demand Theory of the Weak Axiom of Revealed Preference,” *Econometrica: Journal of the Econometric Society*, 44(5), 971–978.

Kim, T. (1987). “Intransitive Indifference and Revealed Preference,” *Econometrica*, 55(1), 163–167.

——— (1991). “The Subjective Expected Utility Hypothesis and Revealed Preference,” *Economic Theory*, 1(1), 251–263.

——— (1996). “Revealed Preference Theory on the Choice of Lotteries,” *Journal of Mathematical Economics*, 26(4), 463–477.

Kim, T., and M. K. Richter (1986). “Nontransitive-Nontotal Consumer Theory,” *Journal of Economic Theory*, 38(2), 324–363.

Kitamura, Y., and J. Stoye (2013). “Nonparametric Analysis of Random Utility Models,” Mimeo, Yale University.

Knoblauch, V. (1992). “A Tight Upper Bound on the Money Metric Utility Function,” *The American Economic Review*, 82(3), 660–663.

——— (1993). “Recovering Homothetic Preferences,” *Economics Letters*, 43(1), 41–45.

——— (2005). “Characterizing Paretian Preferences,” *Social Choice and Welfare*, 25(1), 179–186.

——— (2010). “Recognizing One-Dimensional Euclidean Preference Profiles,” *Journal of Mathematical Economics*, 46(1), 1–5.

Koo, A. Y. C. (1963). “An Empirical Test of Revealed Preference Theory,” *Econometrica*, 31(4), 646–664.

Krauss, E. (1985). “A Representation of Arbitrary Maximal Monotone Operators via Subgradients of Skew-Symmetric Saddle Functions,” *Nonlinear Analysis: Theory, Methods & Applications*, 9(12), 1381–1399.

Krivine, J.-L. (1964). “Anneaux Préordonnés,” *Journal d’Analyse Mathématique*, 12(1), 307–326.

Kubler, F., L. Selden, and X. Wei (2014). “Asset Demand Based Tests of Expected Utility Maximization,” *American Economic Review*, 104(11), 3459–3480.

Kuhn, H., and A. Tucker (1956). *Linear Inequalities and Related Systems.(AM-38)*, Vol. 38. Princeton University Press.

Kumbhakar, S. C., and C. K. Lovell (2003). *Stochastic Frontier Analysis*. Cambridge University Press.

Landsburg, S. E. (1981). “Taste Change in the United Kingdom, 1900–1955,” *Journal of Political Economy*, 89(1), 92–104.

Ledyard, J. O. (1986). “The Scope of the Hypothesis of Bayesian Equilibrium,” *Journal of Economic Theory*, 39(1), 59–82.

Lee, S. (2011). “The Testable Implications of Zero-Sum Games,” *Journal of Mathematical Economics*, 48(1), 39–46.

Little, I. M. (1949). “A Reformulation of the Theory of Consumer’s Behaviour,” *Oxford Economic Papers*, 1(1), 90–99.

Lo, K. C. (2000). "Rationalizability and the Savage Axioms," *Economic Theory*, 15(3), 727–733.

Luce, R. D. (1959). *Individual Choice Behavior: A Theoretical Analysis*. New York: John Wiley and Sons.

Luce, R. D., and P. Suppes (1965). "Preference, Utility, and Subjective Probability," in *Handbook of Mathematical Psychology*, edited by R. D. Luce, R. R. Bush, and E. Galanter, Vol. 3, pp. 249–410. New York: John Wiley and Sons.

Machina, M. J. (1985). "Stochastic Choice Functions Generated from Deterministic Preferences over Lotteries," *The Economic Journal*, 95(379), 575–594.

——— (1989). "Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty," *Journal of Economic Literature*, 27(4), 1622–1668.

Machina, M. J., and D. Schmeidler (1992). "A More Robust Definition of Subjective Probability," *Econometrica*, 60(4), 745–780.

Mantel, R. (1974). "On the Characterization of Aggregate Excess Demand," *Journal of Economic Theory*, 7(3), 348–353.

——— (1976). "Homothetic Preferences and Community Excess Demand Functions," *Journal of Economic Theory*, 12(2), 197–201.

——— (1977). "Implications of Microeconomic Theory for Community Excess Demand Functions," *Frontiers of Quantitative Economics*, 3, 111–126.

Manzini, P., and M. Mariotti (2007). "Sequentially Rationalizable Choice," *American Economic Review*, 97(5), 1824–1839.

Mariotti, M. (2008). "What Kind of Preference Maximization Does the Weak Axiom of Revealed Preference Characterize?," *Economic Theory*, 35(2), 403–406.

Marker, D. (2002). *Model Theory: An Introduction*. New York *et al.*: Springer.

Marshall, M. (2008). *Positive Polynomials and Sums of Squares*. Providence, RI: American Mathematical Society.

Mas-Colell, A. (1977). "On the Equilibrium Price Set of an Exchange Economy," *Journal of Mathematical Economics*, 4(2), 117–126.

——— (1978). "On Revealed Preference Analysis," *Review of Economic Studies*, 45(1), 121–131.

——— (1982). "Revealed Preference after Samuelson," in *Samuelson and Neoclassical Economics*, edited by G. R. Feiwel, pp. 72–82. New York *et al.*: Springer.

Mas-Colell, A., M. Whinston, and J. Green (1995). *Microeconomic Theory*, Vol. 1. New York: Oxford University Press.

Masatlioglu, Y., D. Nakajima, and E. Y. Ozbay (2012). "Revealed Attention," *American Economic Review*, 102(5), 2183–2205.

Masatlioglu, Y., and E. A. Ok (2014). "A Canonical Model of Choice with Initial Endowments," *The Review of Economic Studies*, 81(2), 851–883.

Matzkin, R. L. (1991). "Axioms of Revealed Preference for Nonlinear Choice Sets," *Econometrica*, 59(6), 1779–1786.

Matzkin, R. L., and M. K. Richter (1991). "Testing Strictly Concave Rationality," *Journal of Economic Theory*, 53(2), 287–303.

McFadden, D. (1974). "Conditional Logit Analysis of Qualitative Choice Behavior," in *Frontiers in Econometrics*, edited by P. Zarembka, p. 105. New York: Academic Press.

McFadden, D., A. Mas-Colell, R. Mantel, and M. K. Richter (1974). "A Characterization of Community Excess Demand Functions," *Journal of Economic Theory*, 9(4), 361–374.

McFadden, D., and M. Richter (1971). "On the Extension of a Set Function on a Set of Events to a Probability on the Generated Boolean σ-Algebra," UC Berkeley, working paper.

——— (1990). "Stochastic Rationality and Revealed Stochastic Preference," in *Preferences, Uncertainty, and Optimality, Essays in Honor of Leo Hurwicz*, edited by J. S. Chipman, D. McFadden, and M. K. Richter, pp. 161–186. Boulder, CO: Westview Press Inc.

McGarvey, D. C. (1953). "A Theorem on the Construction of Voting Paradoxes," *Econometrica*, 21(4), 608–610.

Mosteller, F., and P. Nogee (1951). "An Experimental Measurement of Utility," *Journal of Political Economy*, 59(5), 371–404.

Motzkin, T. S. (1936). "Beiträge zur Theorie der linearen Ungleichungen," Dissertation.

Newman, P. (1955). "The Foundations of Revealed Preference Theory," *Oxford Economic Papers*, 7, 151–169.

——— (1960). "Complete Ordering and Revealed Preference," *The Review of Economic Studies*, 27(2), 65–77.

Ok, E. A., P. Ortoleva, and G. Riella (2014). "Revealed (P)reference Theory," *American Economic Review*, forthcoming.

Papadimitriou, C. H., and M. Yannakakis (2010). "An Impossibility Theorem for Price-Adjustment Mechanisms," *Proceedings of the National Academy of Sciences*, 107(5), 1854–1859.

Papandreou, A. G. (1953). "An Experimental Test of an Axiom in the Theory of Choice," *Econometrica*, 21, 477.

Pareto, V. (1906). *Manuale di Economia Politica*. Societa Editrice.

Park, I.-U. (1998). "A Revealed-Preference Implication of Weighted Utility Decisions under Uncertainty," *Economic Theory*, 11, 413–426.

Parrilo, P. A. (2003). "Semidefinite Programming Relaxations for Semialgebraic Problems," *Mathematical Programming*, 96(2), 293–320.

——— (2004). "Sum of Squares Programs and Polynomial Inequalities," in *SIAG/OPT Views-and-News: A Forum for the SIAM Activity Group on Optimization*, Vol. 15, pp. 7–15.

Peleg, B., and S. Tijs (1996). "The Consistency Principle for Games in Strategic Form," *International Journal of Game Theory*, 25(1), 13–34.

Peters, H. J., and P. Wakker (1994). "WARP Does not Imply SARP for More than Two Commodities," *Journal of Economic Theory*, 62(1), 152–160.

——— (1996). "Cycle-Preserving Extension of Demand Functions to New Commodities," *Journal of Mathematical Economics*, 25(3), 281–290.

Pfanzagl, J., V. Baumann, and H. Huber (1971). *Theory of Measurement*. Heidelberg, Germany: Physica-Verlag.

Plott, C. R. (1974). "On Game Solutions and Revealed Preference Theory," working paper.

Poincaré, H. (1895). "Complément a l'Analysis Situs," *Rendiconti del Circolo Matematico di Palermo*, 13, 285–343.

Polisson, M., and J. Quah (2013). "Revealed Preference in a Discrete Consumption Space," *American Economic Journal: Microeconomics*, 5(1), 28–34.

Polisson, M., L. Renou, and J. Quah (2013). "Revealed Preference Tests under Risk and Uncertainty," working paper 13/24, University of Leicester.

Popper, K. R. (1959). *The Logic of Scientific Discovery*. London: Hutchinson.

Quah, J. K.-H. (2013). "A Revealed Preference Test for Weakly Separable Preferences," working paper, University of Oxford.

Quah, J. K.-H., H. Nishimura, and E. A. Ok (2013). "A Unified Approach to Revealed Preference Theory: The Case of Rational Choice," Mimeo, University of Oxford.

Rader, T. (1963). "The Existence of a Utility Function to Represent Preferences," *The Review of Economic Studies*, 30(3), 229–232.

Ray, I., and L. Zhou (2001). "Game Theory via Revealed Preferences," *Games and Economic Behavior*, 37(2), 415–424.

Ray, S. C., and D. Bhadra (1993). "Nonparametric Tests of Cost Minimizing Behavior: A Study of Indian Farms," *American Journal of Agricultural Economics*, 75(4), 990–999.

Rehbeck, J. (2014). "Every Choice Correspondence is Backwards-Induction Rationalizable," *Games and Economic Behavior*, 88, 207–210.

Reny, P. (2014). "A Characterization of Rationalizable Consumer Behavior," *Econometrica*, forthcoming.

Richter, M. K. (1966). "Revealed Preference Theory," *Econometrica*, 34(3), 635–645.

——— (1971). "Rational Choice," in *Preferences, Utility and Demand*, edited by J. S. Chipman, L. Hurwicz, M. K. Richter, and H. F. Sonnenschein, pp. 29–58. Orlando, FL: Harcourt Brace Jovanivic Inc.

——— (1975). "Rational Choice and Polynomial Measurement Models," *Journal of Mathematical Psychology*, 12(1), 99–113.

Richter, M. K. (1979). "Duality and Rationality," *Journal of Economic Theory*, 20(2), 131–181.

Richter, M. K., and L. Shapiro (1978). "Revelations of a Gambler," *Journal of Mathematical Economics*, 5(3), 229–244.

Richter, M. K., and K.-C. Wong (2004). "Concave Utility on Finite Sets," *Journal of Economic Theory*, 115(2), 341–357.

Rochet, J. (1987). "A Necessary and Sufficient Condition for Rationalizability in a Quasi-Linear Context," *Journal of Mathematical Economics*, 16, 191–200.

Rockafellar, R. T. (1966). "Characterization of the Subdifferentials of Convex Functions," *Pacific Journal of Mathematics*, 17(3), 497–510.

——— (1997). *Convex Analysis*. Princeton University Press.

Rose, H. (1958). "Consistency of Preference: The Two-Commodity Case," *The Review of Economic Studies*, 25(2), 124–125.

Rota, G.-C. (1964). "On the Foundations of Combinatorial Theory I. Theory of Möbius Functions," *Probability Theory and Related Fields*, 2(4), 340–368.

Rubin, D. B. (1973). "Matching to Remove Bias in Observational Studies," *Biometrics*, 29(1), 159–183.

Rudin, W. (1976). *Principles of Mathematical Analysis*. New York: McGraw-Hill.

Rynasiewicz, R. A. (1983). "Falsifiability and the Semantic Eliminability of Theoretical Languages," *The British Journal for the Philosophy of Science*, 34(3), 225–241.

Samuelson, P. A. (1938). "A Note on the Pure Theory of Consumer's Behaviour," *Economica*, 5(17), 61–71.

——— (1948). "Consumption Theory in Terms of Revealed Preference," *Economica*, 15(60), 243–253.

——— (1950). "The Problem of Integrability in Utility Theory," *Economica*, 17(68), 355–385.

——— (1963). "Problems of Methodology—Discussion," *American Economic Review*, 53, 232–236.

——— (1964). "Theory and Realism: A Reply," *American Economic Review*, 54, 736–739.

——— (1965). "Professor Samuelson on Theory and Realism: Reply," *American Economic Review*, 55, 1162–1172.

Savage, L. J. (1954). *The Foundations of Statistics*. New York: John Wiley and Sons.

Schipper, B. C. (2009). "How Mindless is Standard Economics Really?" Working papers, University of California, Department of Economics.

Schrijver, A. (1998). *Theory of Linear and Integer Programming*. New York: John Wiley & Sons.

Scott, D. (1964). "Measurement Structures and Linear Inequalities," *Journal of Mathematical Psychology*, 1(2), 233–247.

Seidenberg, A. (1954). "A New Decision Method for Elementary Algebra," *The Annals of Mathematics*, 60(2), 365–374.

Selten, R. (1991). "Properties of a Measure of Predictive Success," *Mathematical Social Sciences*, 21(2), 153–167.

Sen, A. (1969). "Quasi-Transitivity, Rational Choice and Collective Decisions," *The Review of Economic Studies*, 36(3), 381–393.

——— (1971). "Choice Functions and Revealed Preference," *The Review of Economic Studies*, 38(3), 307–317.

Shafer, W. (1977). "Revealed Preference Cycles and the Slutsky Matrix," *Journal of Economic Theory*, 16(2), 293–309.

Shafer, W., and H. Sonnenschein (1982). "Market Demand and Excess Demand Functions," in *Handbook of Mathematical Economics*, edited by K. J. Arrow, W. Hildenbrand, M. D. Intriligator, and H. Sonnenschein, Vol. 2, pp. 671–693. Amsterdam: North-Holland.

Shen, W.-M., and H. A. Simon (1993). "Fitness Requirements for Scientific Theories Containing Recursive Theoretical Terms," *The British Journal for the Philosophy of Science*, 44(4), 641–652.

Shirai, K. (2010). "On the Existence of a Submodular Utility Function," Mimeo, Waseda University.

Shmaya, E., and L. Yariv (2012). "Experiments on Decisions Under Uncertainty: A Theoretical Framework," working paper.

Simon, H. A. (1955). "A Behavioral Model of Rational Choice," *The Quarterly Journal of Economics*, 69(1), 99.

——— (1979). "Fit, Finite, and Universal Axiomatization of Theories," *Philosophy of Science*, 46(2), 295–301.

——— (1983). "Fitness Requirements for Scientific Theories," *The British Journal for the Philosophy of Science*, 34(4), 355–365.

——— (1985). "Quantification of Theoretical Terms and the Falsifiability of Theories," *The British Journal for the Philosophy of Science*, 36(3), 291–298.

Simon, H. A., and G. J. Groen (1973). "Ramsey Eliminability and the Testability of Scientific Theories," *The British Journal for the Philosophy of Science*, 24(4), 367–380.

Sippel, R. (1997). "An Experiment on the Pure Theory of Consumer's Behaviour," *The Economic Journal*, 107(444), 1431–1444.

Slater, M. L. (1951). "A Note on Motzkin's Transposition Theorem," *Econometrica*, 19(2), 185–187.

Smeulders, B., L. Cherchye, B. De Rock, and F. C. R. Spieksma (2013). "The Money Pump as a Measure of Revealed Preference Violations: A Comment," *Journal of Political Economy*, 121(6), 1248–1258.

Sneed, J. (1971). *The Logical Structure of Mathematical Physics*. Dordrecht: D. Reidel.

Snyder, S. (1999). "Testable Restrictions of Pareto Optimal Public Good Provision," *Journal of Public Economics*, 71(1), 97–119.

Sondermann, D. (1982). "Revealed Preference: An Elementary Treatment," *Econometrica*, 50(3), 777–779.

Sonnenschein, H. (1972). "Market Excess Demand Functions," *Econometrica*, 40(3), 549–563.

Sprumont, Y. (2000). "On the Testable Implications of Collective Choice Theories," *Journal of Economic Theory*, 93, 205–232.

——— (2001). "Paretian Quasi-Orders: The Regular Two-Agent Case," *Journal of Economic Theory*, 101(2), 437–456.

Stegmüller, W., W. Balzer, and W. Spohn (1982). *Philosophy of Economics: Proceedings, Munich, July 1981*. Berlin *et al.*: Springer-Verlag.

Stengle, G. (1974). "A Nullstellensatz and a Positivstellensatz in Semialgebraic Geometry," *Mathematische Annalen*, 207(2), 87–97.

Stigum, B. P. (1973). "Revealed Preference—A Proof of Houthakker's Theorem," *Econometrica*, 41(3), 411–423.

Stoer, J., and C. Witzgall (1970). *Convexity and Optimization in Finite Dimensions*. Berlin *et al.*: Springer-Verlag.

Sturmfels, B. (2002). *Solving Systems of Polynomial Equations*. Providence, RI: American Mathematical Society.

Suzumura, K. (1976a). "Rational Choice and Revealed Preference," *The Review of Economic Studies*, 43(1), 149–158.

——— (1976b). "Remarks on the Theory of Collective Choice," *Economica*, 43(172), 381–390.

——— (1977). "Houthakker's Axiom in the Theory of Rational Choice," *Journal of Economic Theory*, 14(2), 284–290.

Swofford, J. L., and G. A. Whitney (1986). "Flexible Functional Forms and the Utility Approach to the Demand for Money: A Nonparametric Analysis: Note," *Journal of Money, Credit and Banking*, 18(3), 383–389.

——— (1987). "Nonparametric Tests of Utility Maximization and Weak Separability for Consumption, Leisure and Money," *The Review of Economics and Statistics*, 69(3), 458–464.

Szpilrajn, E. (1930). "Sur l'Extension de l'Ordre Partiel," *Fundamenta Mathematicae*, 16, 386–389.

Tarski, A. (1951). "A Decision Method for Elementary Algebra and Geometry (2nd ed., revised)," Berkeley and Los Angeles: Rand Corporation monograph.

——— (1954). "Contributions to the Theory of Models I," *Indagationes Mathematicae*, 16, 572–581.

Tauer, L. W. (1995). "Do New York Dairy Farmers Maximize Profits or Minimize Costs?" *American Journal of Agricultural Economics*, 77(2), 421–429.

Tversky, A. (1967). "A General Theory of Polynomial Conjoint Measurement," *Journal of Mathematical Psychology*, 4, 1–20.

Tyson, C. (2008). "Cognitive Constraints, Contraction Consistency, and the Satisficing Criterion," *Journal of Economic Theory*, 138(1), 51–70.

Uzawa, H. (1956). "Note on Preference and Axioms of Choice," *Annals of the Institute of Statistical Mathematics*, 8(1), 35–40.

——— (1960a). "Preference and Rational Choice in the Theory of Consumption," in *Mathematical Methods in the Social Sciences, 1959: Proceedings of the First Stanford Symposium*, edited by K. J. Arrow, S. Karlin, and P. Suppes, pp. 129–150. Palo Alto, CA: Stanford University Press.

——— (1960b). "Walras' Tâtonnement in the Theory of Exchange," *The Review of Economic Studies*, 27(3), 182–194.

——— (1971). "Preference and Rational Choice in the Theory of Consumption," in *Preferences, Utility, and Demand: A Minnesota Symposium*, edited by J. S. Chipman, L. Hurwicz, M. K. Richter, and H. F. Sonnenschein, pp. 7–28. Orlando, FL: Harcourt Brace Jovanovich, Inc.

Van Benthem, J. (1976). "A Problem Concerning Expansions and its Connections with Modal Logic," working paper.

Varian, H. R. (1982). "The Nonparametric Approach to Demand Analysis," *Econometrica*, 50(4), 945–974.

——— (1983a). "Non-Parametric Tests of Consumer Behaviour," *Review of Economic Studies*, 50(1), 99–110.

——— (1983b). "Nonparametric Tests of Models of Investor Behavior," *Journal of Financial and Quantitative Analysis*, 18(3), 269–278.

——— (1984). "The Nonparametric Approach to Production Analysis," *Econometrica*, 52(3), 579–598.

——— (1985). "Non-Parametric Analysis of Optimizing Behavior with Measurement Error," *Journal of Econometrics*, 30, 445–458.

——— (1988a). "Revealed Preference with a Subset of Goods," *Journal of Economic Theory*, 46(1), 179–185.

——— (1988b). "Estimating Risk Aversion from Arrow-Debreu Portfolio Choice," *Econometrica*, 43(4), 973–979.

——— (1990). "Goodness-of-Fit in Optimizing Models," *Journal of Econometrics*, 46(1–2), 125–140.

——— (1991). "Goodness-of-Fit for Revealed Preference Tests," working paper, Department of Economics, University of Michigan.

——— (2006). "Revealed Preference," in *Samuelsonian Economics and the Twenty-First Century*, edited by M. Szenberg and L. Ramrattan, pp. 99–115. Oxford University Press.

Veblen, O., and J. Alexander (1912–1913). “Manifolds of *n* Dimensions,” *Annals of Mathematics*, 14(1/2), 163–178.

Ville, J. (1946). “Sur les Conditions d’Existence d’une Ophélimité Totale et d’un Indice du Niveau des Prix,” *Annales de L’Université de Lyon*, 9, 32–39.

Ville, J., and P. Newman (1951–1952). “The Existence Conditions of a Total Utility Function,” *Review of Economic Studies*, 19(2), 123–128.

Von Neumann, J. (1953). “A Certain Zero-Sum Two-Person Game Equivalent to the Optimal Assignment Problem,” *Contributions to the Theory of Games*, 2, 5–12.

Wald, A. (1947a). “An Essentially Complete Class of Admissible Decision Functions,” *The Annals of Mathematical Statistics*, 18(4), 549–555.

——— (1947b). “Foundations of a General Theory of Sequential Decision Functions,” *Econometrica*, 15(4), 279–313.

——— (1950). *Statistical Decision Functions*. New York: John Wiley and Sons.

Wilson, R. (1970). “The Finer Structure of Revealed Preference,” *Journal of Economic Theory*, 2(4), 348–353.

Xiong, S. (2013). “Every Choice Correspondence is Backwards-Induction Rationalizable,” working paper.

Xu, Y., and L. Zhou (2007). “Rationalizability of Choice Functions by Game Trees,” *Journal of Economic Theory*, 134(1), 548–556.

Yannakakis, M. (1982). “The Complexity of the Partial Order Dimension Problem,” *SIAM Journal on Algebraic and Discrete Methods*, 3, 351.

Yanovskaya, E. (1980). “Revealed Preference in Noncooperative Games” (in Russian), *Mathematical Methods of Social Science*, 13, 73–81.

Yatchew, A. (1985). “A Note on Non-Parametric Tests of Consumer Behaviour,” *Economics Letters*, 18(1), 45–48.

Yellott, J. I. (1977). “The Relationship between Luce’s Choice Axiom, Thurstone’s Theory of Comparative Judgment, and the Double Exponential Distribution,” *Journal of Mathematical Psychology*, 15(2), 109–144.

Index

Other titles in the series (*continued fron page iii*)

A. Colin Cameron and Pravin K. Trivedi, *Regression analysis of count data*, 9780521632010, 9780521635677

Steinar Strom, Editor, *Econometrics and economic theory in the 20th century: The Ragnar Frisch Centennial Symposium*, 9780521633239, 9780521633659

Eric Ghysels, Norman R. Swanson, and Mark Watson, Editors, *Essays in econometrics: Collected papers of Clive W. J. Granger (Volume I)*, 9780521772976, 9780521774963

Eric Ghysels, Norman R. Swanson, and Mark Watson, Editors, *Essays in econometrics: Collected papers of Clive W. J. Granger (Volume II)*, 9780521792073, 9780521796491

Cheng Hsiao, *Analysis of panel data, second edition*, 9780521818551, 9780521522717

Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, Editors, *Advances in economics and econometrics – Eighth World Congress (Volume I)*, 9780521818728, 9780521524117

Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, Editors, *Advances in economics and econometrics – Eighth World Congress (Volume II)*, 9780521818735, 9780521524124

Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, Editors, *Advances in economics and econometrics – Eighth World Congress (Volume III)*, 9780521818742, 9780521524131

Roger Koenker, *Quantile regression*, 9780521845731, 9780521608275

Charles Blackorby, Walter Bossert, and David Donaldson, *Population issues in social choice theory, welfare economics, and ethics*, 9780521825511, 9780521532587

John E. Roemer, *Democracy, education, and equality*, 9780521846653, 9780521609135

Richard Blundell, Whitney K. Newey, and Thorsten Persson, *Advances in economics and econometrics – Ninth World Congress (Volume I)*, 9780521871525, 9780521692083

Richard Blundell, Whitney K. Newey, and Thorsten Persson, *Advances in economics and econometrics – Ninth World Congress (Volume II)*, 9780521871532, 9780521692090

Richard Blundell, Whitney K. Newey, and Thorsten Persson, *Advances in economics and econometrics – Ninth World Congress (Volume III)*, 9780521871549, 9780521692106

Fernando Vega-Redondo, *Complex social networks*, 9780521857406, 9780521674096

Itzhak Gilboa, *Theory of decision under uncertainty*, 9780521517324, 9780521741231

Krislert Samphantharak and Robert M. Townsend, *Households as corporate firms: An analysis of household finance using integrated household surveys and corporate financial accounting*, 9780521195829, 9780521124164

Rakesh Vohra, *Mechanism design: A linear programming approach*, 9781107004368, 9780521179461

Daron Acemoglu, Manuel Arellano, Eddie Dekel, *Advances in economics and econometrics – Tenth World Congress (Volume I)*, 9781107016040, 9781107638105

Daron Acemoglu, Manuel Arellano, Eddie Dekel, *Advances in economics and econometrics – Tenth World Congress (Volume II)*, 9781107016057, 9781107674165

Daron Acemoglu, Manuel Arellano, Eddie Dekel, *Advances in economics and econometrics – Tenth World Congress (Volume III)*, 9781107016064, 9781107627314

Andrew Harvey, *Dynamic models for volatility and heavy tails: With applications to financial and economic time series*, 9781107034723, 9781107630024

Cheng Hsiao, *An analysis of panel data* (second edition), 9781107038691, 9781107657632

Jean-François Mertens, Sylvain Sorin, Shmuel Zamir, *Repeated games,* 9781107030206, 9781107662636